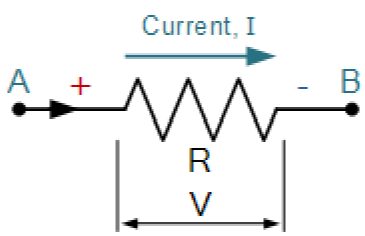
Basic Laws

Kirchhoff’s Voltage Law

Kirchhoff’s Voltage Law states that for a closed loop series path **the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero**.

The algebraic sum of ALL the potential differences around the loop must be equal to zero as: $\Sigma V = 0$. Note here that the term “algebraic sum” means to take into account the polarities and signs of the sources and voltage drops around the loop.

A Single Circuit Element



For this simple example we will assume that the current, I is in the same direction as the flow of positive charge, that is conventional current flow.

Here the flow of current through the resistor is from point A to point B, that is from positive terminal to a negative terminal.

Thus as we are travelling in the same direction as current flow, there will be a *fall* in potential across the resistive element giving rise to a -IR voltage drop across it.

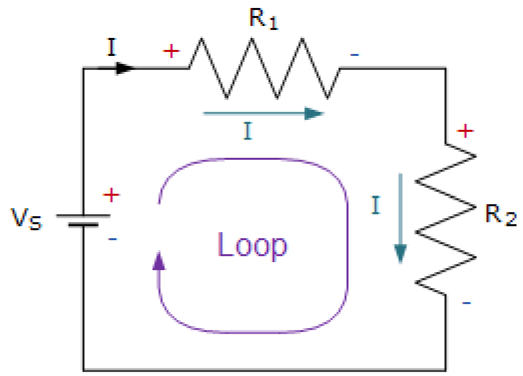
If the flow of current was in the opposite direction from point B to point A, then there would be a *rise* in potential across the resistive element as we are moving from a - potential to a + potential giving us a +IR voltage drop.

Thus to apply Kirchhoff’s voltage law correctly to a circuit, we must first understand the direction of the polarity and as we can see, the sign of the voltage drop across the resistive element will depend on the direction of the current flowing through it. As a general rule, you will loose potential in the same direction of current across an element and gain potential as you move in the direction of an emf source.

The direction of current flow around a closed circuit can be assumed to be either clockwise or anticlockwise and either one can be chosen. If the direction chosen is different from the actual direction of current flow, the result will still be correct and valid but will result in the algebraic answer having a minus sign.

To understand this idea a little more, lets look at a single circuit loop to see if Kirchhoff’s Voltage Law holds true.

A Single Circuit Loop



Kirchhoff's voltage law states that the algebraic sum of the potential differences in any loop must be equal to zero as: ΣV = 0. Since the two resistors, R_1 and R_2 are wired together in a series connection, they are both part of the same loop so the same current must flow through each resistor.

Thus the voltage drop across resistor, $R_1 = I*R_1$ and the voltage drop across resistor, $R_2 = I*R_2$ giving by KVL:

$$V_S + (-IR_1) + (-IR_2) = 0$$

$$\therefore V_S = IR_1 + IR_2$$

$$V_S = I(R_1 + R_2)$$

$$V_S = IR_T$$

$$\text{Where: } R_T = R_1 + R_2$$

We can see that applying Kirchhoff's Voltage Law to this single closed loop produces the formula for the equivalent or total resistance in the series circuit and we can expand on this to find the values of the voltage drops around the loop.

$$R_T = R_1 + R_2$$

$$I = \frac{V_S}{R_T} = \frac{V_S}{R_1 + R_2}$$

$$V_{R1} = IR_1 = V_S\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_{R2} = IR_2 = V_S\left(\frac{R_2}{R_1 + R_2}\right)$$

Example 1

Three resistor of values: 10 ohms, 20 ohms and 30 ohms, respectively are connected in series across a 12 volt battery supply. Calculate: a) the total resistance, b) the circuit current, c) the current through each resistor, d) the voltage drop across each resistor, e) verify that Kirchhoff's voltage law, KVL holds true.

a) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3 = 10\Omega + 20\Omega + 30\Omega = 60\Omega$$

Then the total circuit resistance R_T is equal to 60Ω

b) Circuit Current (I)

$$I = \frac{V_S}{R_T} = \frac{12}{60} = 0.2A$$

Thus the total circuit current I is equal to 0.2 amperes or 200mA

c) Current Through Each Resistor

The resistors are wired together in series, they are all part of the same loop and therefore each experience the same amount of current. Thus:

$$I_{R1} = I_{R2} = I_{R3} = I_{SERIES} = 0.2 \text{ amperes}$$

d) Voltage Drop Across Each Resistor

$$V_{R1} = I \times R_1 = 0.2 \times 10 = 2 \text{ volts}$$

$$V_{R2} = I \times R_2 = 0.2 \times 20 = 4 \text{ volts}$$

$$V_{R3} = I \times R_3 = 0.2 \times 30 = 6 \text{ volts}$$

e) Verify Kirchhoff's Voltage Law

$$V_S + (-IR_1) + (-IR_2) + (-IR_3) = 0$$

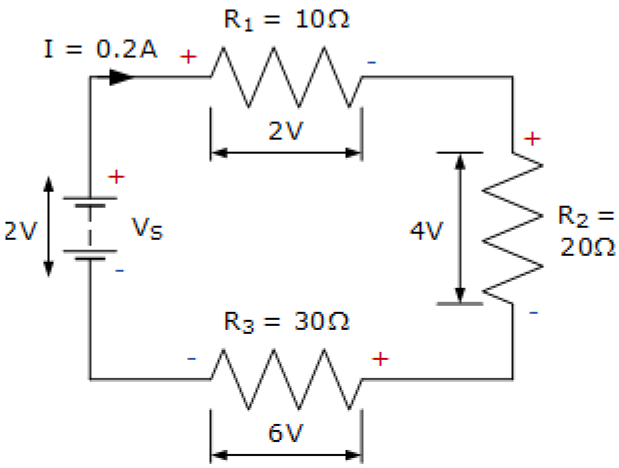
$$12 + (-0.2\times10) + (-0.2\times20) + (-0.2\times30) = 0$$

$$12 + (-2) + (-4) + (-6) = 0$$

$$\therefore 12 - 2 - 4 - 6 = 0$$

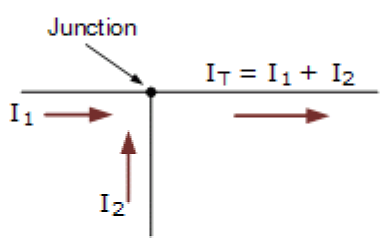
Thus Kirchhoff's voltage law holds true as the individual voltage drops around the closed loop add up to the total.

Kirchhoff's Circuit Loop



Kirchhoff's Current Law

Kirchhoff's current law (KCL) states that the algebraic sum of the currents at a node is zero. Or the sum of all currents entering a node equals to the sum of all currents leaving it.

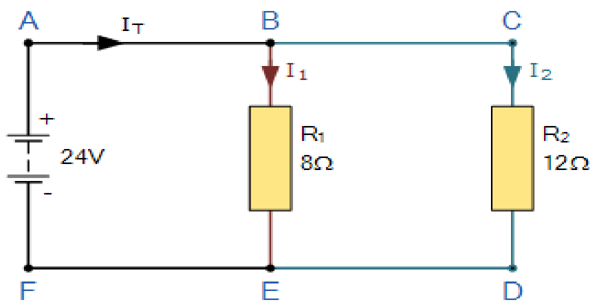


Here in this simple single junction example, the current I_T leaving the junction is the algebraic sum of the two currents, I_1 and I_2 entering the same junction. That is $I_T = I_1 + I_2$.

Note that we could also write this correctly as the algebraic sum of: $I_T - (I_1 + I_2) = 0$.

Resistors in Parallel

Let's look how we could apply Kirchhoff's current law to resistors in parallel, whether the resistances in those branches are equal or unequal. Consider the following circuit diagram:



In this simple parallel resistor example there are two distinct junctions for current. Junction one occurs at node B, and junction two occurs at node E. Thus we can use Kirchhoff's Junction Rule for the electrical currents at both of these two distinct junctions, for those currents entering the junction and for those currents flowing leaving the junction.

For current branch B to E through resistor R_1

$$I_1 = \frac{V}{R_1} = \frac{24}{8} = 3A$$

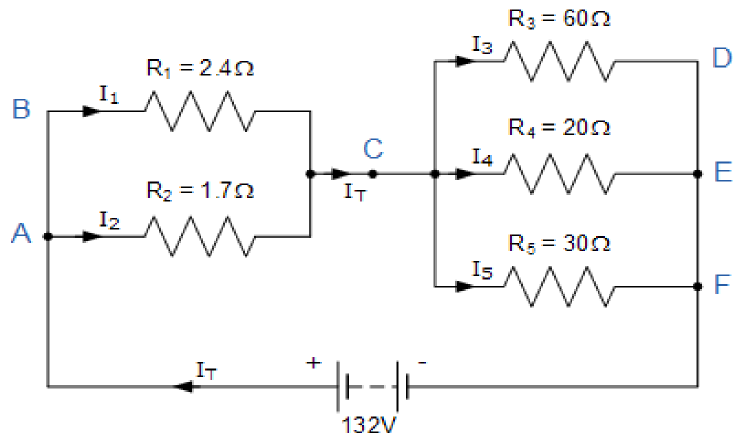
For current branch C to D through resistor R_2

$$I_2 = \frac{V}{R_2} = \frac{24}{12} = 2A$$

Applying KCL to more complex circuits.

We can use Kirchhoff's current law to find the currents flowing around more complex circuits. We hopefully know by now that the algebraic sum of all the currents at a node (junction point) is equal to zero and with this idea in mind, it is a simple case of determining the currents entering a node and those leaving the node. Consider the circuit below.

Example 2



Circuit Resistance R_{AC}

$$\frac{1}{R_{(AC)}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2.4} + \frac{1}{1.7}$$

$$\frac{1}{R_{(AC)}} = \frac{1}{1} \qquad \therefore R_{(AC)} = 1\Omega$$

Thus the equivalent circuit resistance between nodes A and C is calculated as 1 Ohm.

Circuit Resistance R_{CF}

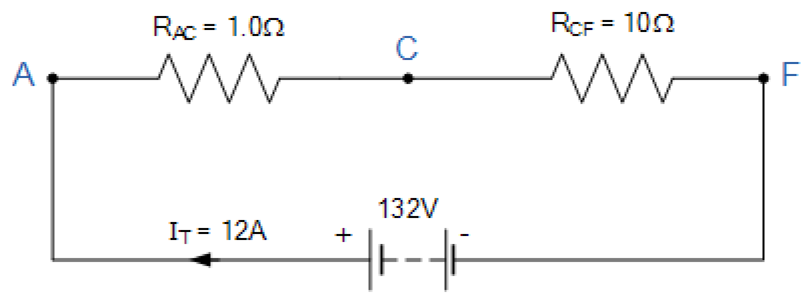
Thus the equivalent circuit resistance between nodes C and F is calculated as 10 Ohms. Then the total circuit current, I_T is given as:

$$R_T = R_{(AC)} + R_{(CF)} = 1 + 10 = 11\Omega$$

$$I_T = \frac{V}{R_T} = \frac{132}{11} = 12\,\text{Amps}$$

Giving us an equivalent circuit of:

Kirchhoff's Current Law Equivalent Circuit



Therefore, V = 132V, R_{AC} = 1Ω, R_{CF} = 10Ω's and I_T = 12A.

Having established the equivalent parallel resistances and supply current, we can now calculate the individual branch currents and confirm using Kirchhoff's junction rule as follows.

$$V_{AC} = I_T \times R_{AC} = 12 \times 1 = 12 \text{ Volts}$$

$$V_{CF} = I_T \times R_{CF} = 12 \times 10 = 120 \text{ Volts}$$

$$I_1 = \frac{V_{AC}}{R_1} = \frac{12}{2.4} = 5 \text{ Amps}$$

$$I_2 = \frac{V_{AC}}{R_2} = \frac{12}{1.7} = 7 \text{ Amps}$$

$$I_3 = \frac{V_{CF}}{R_3} = \frac{120}{60} = 2 \text{ Amps}$$

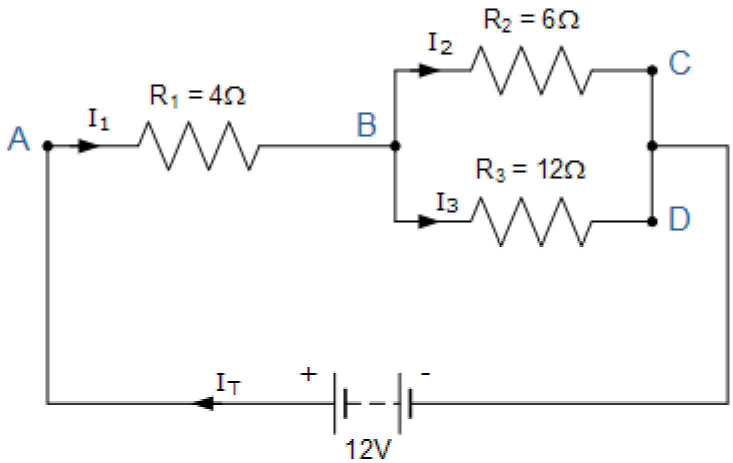
$$I_4 = \frac{V_{CF}}{R_4} = \frac{120}{20} = 6 \text{ Amps}$$

$$I_5 = \frac{V_{CF}}{R_5} = \frac{120}{30} = 4 \text{ Amps}$$

Thus, I_1 = 5A, I_2 = 7A, I_3 = 2A, I_4 = 6A, and I_5 = 4A.

Example 3

Find the currents flowing around the following circuit using Kirchhoff's Current Law only.



Kirchhoff's Loop Equations

Loop (ABC)	Loop (ABD)
$12 = 4I_1 + 6I_2$	$12 = 4I_1 + 12I_3$
$12 = 4(I_2 + I_3) + 6I_2$	$12 = 4(I_2 + I_3) + 12I_3$
$12 = 4I_2 + 4I_3 + 6I_2$	$12 = 4I_2 + 4I_3 + 12I_3$
$12 = 10I_2 + 4I_3$	$12 = 4I_2 + 16I_3$

We now have two simultaneous equations that relate to the currents flowing around the circuit.

Eq. No 1 : $12 = 10I_2 + 4I_3$

Eq. No 2 : $12 = 4I_2 + 16I_3$

By multiplying the first equation (Loop ABC) by 4 and subtracting Loop ABD from Loop ABC, we can be reduced both equations to give us the values of I_2 and I_3

Eq. No 1 : $12 = 10I_2 + 4I_3$ (x4) $\Rightarrow$ $48 = 40I_2 + 16I_3$

Eq. No 2 : $12 = 4I_2 + 16I_3$ (x1) $\Rightarrow$ $12 = 4I_2 + 16I_3$

Eq. No 1 – Eq. No 2 $\Rightarrow$ $36 = 36I_2 + 0$

Substitution of I_2 in terms of I_3 gives us the value of I_2 as 1.0 Amps

Now we can do the same procedure to find the value of I_3 by multiplying the first equation (Loop ABC) by 4 and the second equation (Loop ABD) by 10. Again by subtracting Loop ABC from Loop ABD, we can be reduced both equations to give us the values of I_2 and I_3

Eq. No 1 : $12 = 10I_2 + 4I_3$ (x4) $\Rightarrow$ $48 = 40I_2 + 16I_3$

Eq. No 2 : $12 = 4I_2 + 16I_3$ (x10) $\Rightarrow$ $120 = 40I_2 + 160I_3$

Eq. No 2 – Eq. No 1 $\Rightarrow$ $72 = 0 + 144I_3$

Thus substitution of I_3 in terms of I_2 gives us the value of I_3 as 0.5 Amps

As Kirchhoff's junction rule states that : $I_1 = I_2 + I_3$

The supply current flowing through resistor R_1 is given as : $1.0 + 0.5 = 1.5$ Amps

Thus $I_1 = I_T = 1.5$ Amps, $I_2 = 1.0$ Amps and $I_3 = 0.5$ Amps and from that information we could calculate the I*R voltage drops across the devices and at the various points (nodes) around the circuit.

Common DC Circuit Theory Terms:

- Circuit – a circuit is a closed loop conducting path in which an electrical current flows.
- Path – a single line of connecting elements or sources.
- Node – a node is a junction, connection or terminal within a circuit were two or more circuit elements are connected or joined together giving a connection point between two or more branches. A node is indicated by a dot.
- Branch – a branch is a single or group of components such as resistors or a source which are connected between two nodes.
- Loop – a loop is a simple closed path in a circuit in which no circuit element or node is encountered more than once.

• Mesh – a mesh is a single closed loop series path that does not contain any other paths. There are no loops inside a mesh.

Note that:

Components are said to be connected together in Series if the same current value flows through all the components.

Components are said to be connected together in Parallel if they have the same voltage applied across them.

Circuit Elements in Series

A simplified series circuit is made up of three elements:

- a source,
- resistors, in which the electrical energy is utilized and is commonly referred to as loads,
- and ideal conductors, with no assumed resistance, to connect the elements in series.

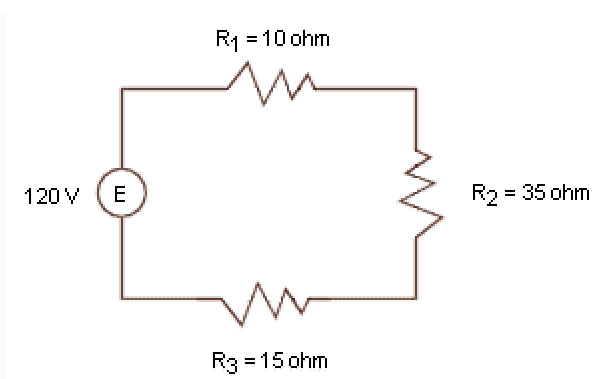
Since there is only one path for current to flow in this series circuit, the current or electron flow must be the same in each segment of the circuit. This means that the current leaving the source is equal to the amount of current through each resistance.

There are three rules governing the simple series circuits of resistive elements. They are:

1. The current flow is the same through each element of the series circuit.
2. The combined resistance of the various loads in series is the sum of the separate resistances.
3. The voltage across the source or power supply is equal to the sum of the voltage drops across the separate loads in series.

> The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

Example



Applying Ohm's Law for the total circuit, we find

$$R_T = R_1 + R_2 + R_3 = 10\text{ ohm} + 35\text{ ohm} + 15\text{ ohm} = 60\text{ ohm}$$

$$I_T = E_T / R_T = \frac{120\text{ Volts}}{60\text{ ohm}} = 2\text{ amps}$$

$$I_T = I_1 = I_2 = I_3 = 2\text{ amps}$$

$$E_1 = I_1 R_1 = 2\text{ amps x } 10\text{ ohms} = 20\text{ volts}$$

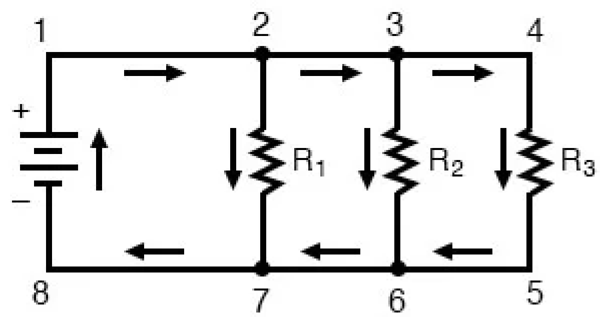
$$E_2 = I_2 R_2 = 2\text{ amps x } 35\text{ ohms} = 70\text{ volts}$$

$$E_3 = I_3 R_3 = 2\text{ amps x } 15\text{ ohms} = 30\text{ volts}$$

$$E_T = E_1 + E_2 + E_3$$

$$120\text{ volts} = 20\text{ volts} + 70\text{ volts} + 30\text{ volts}$$

Circuit Elements in Parallel



For three circuit elements connected in parallel as shown in Fig. above, KCL states that the current i entering the principal node is the sum of the three currents leaving the node through the branches.

$$i = i_1 + i_2 + i_3$$

For the three passive circuit elements of resistances,

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v = \frac{1}{R_{eq}}\,v$$

For several resistors in parallel,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$ ……………eq. 2.38

The equivalent resistance of two resistors in parallel is given by the product over the sum of the two resistors.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. From Eq. (2.38), the equivalent conductance for N resistors in parallel is

$$\boxed{G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N}$$ ……….eq.2.40

where $G_{eq} = 1/R_{eq}, G_1 = 1/R_1, G_2 = 1/R_2, G_3 = 1/R_3, \ldots, G_N = 1/R_N$. Equation (2.40) states:

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

- Combinations of inductances in parallel have similar expressions to those of resistors in parallel:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \qquad \text{and, for two inductances,} \qquad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Example: Obtain the equivalent resistance of (a) two 60 Ω resistors in parallel and (b) three 60.Ω resistors in parallel

(a) $$R_{eq} = \frac{(60.0)^2}{120.0} = 30.0\,\Omega$$

(b) $$\frac{1}{R_{eq}} = \frac{1}{60.0} + \frac{1}{60.0} + \frac{1}{60.0} \qquad R_{eq} = 20.0\,\Omega$$

EXAMPLE. Two inductances L1 = 3:0mH and L2 = 6:0mH are connected in parallel. Find Leq.

$$\frac{1}{L_{eq}} = \frac{1}{3.0\,\text{mH}} + \frac{1}{6.0\,\text{mH}} \qquad \text{and} \qquad L_{eq} = 2.0\,\text{mH}$$

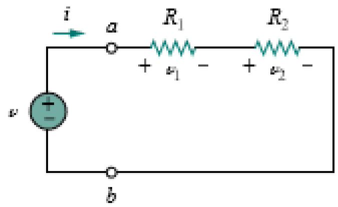
- **With three capacitances in parallel,**

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} = (C_1 + C_2 + C_3) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

For several parallel capacitors, Ceq = C1 + C2 + ……, which is of the same form as resistors in series.

Voltage Division

A set of series-connected resistors as shown in Fig. below is referred to as a voltage divider.



To determine the voltage across each resistor in the above Fig.,

$$\boxed{v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v}$$

Since, $v_1 = iR_1, \qquad v_2 = iR_2$ and

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$i = \frac{v}{R_1 + R_2}$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 2.29 is called a voltage divider. In general, if a voltage divider has N resistors (R1,R2,...,RN) in series with the source voltage v, the nth resistor (Rn) will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Current Division

A parallel arrangement of resistors as shown in Fig. below results in a current divider. The ratio of the branch current i1 to the total current i illustrates the operation of the divider.

We know that the equivalent resistor has the same voltage, or

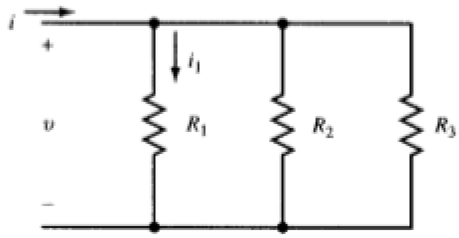


Fig. 3-6

$$i = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \qquad \text{and} \qquad i_1 = \frac{v}{R_1}$$

$$\frac{i_1}{i} = \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3} = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

which shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig. 2.36 is known as a current divider. Notice that the larger current flows through the smaller resistance.

If a current divider has N conductors (G1,G2,...,GN) in parallel with the source current i, the nth conductor (Gn) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} i$$

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single equivalent resistance Req. Such an equivalent resistance is the resistance between the designated terminals of the network and must exhibit the same i-v characteristics as the original network at the terminals.